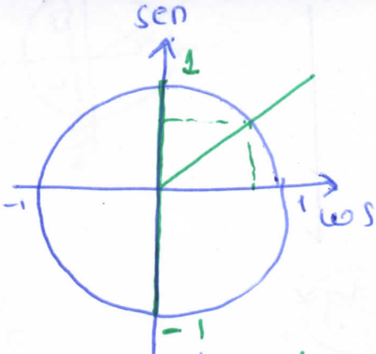
EQUAÇÕES TRIGONOMÉTRICAS

$\operatorname{sen} x = b$

$-1 \leq b \leq 1$

$\operatorname{sen} x = 2$

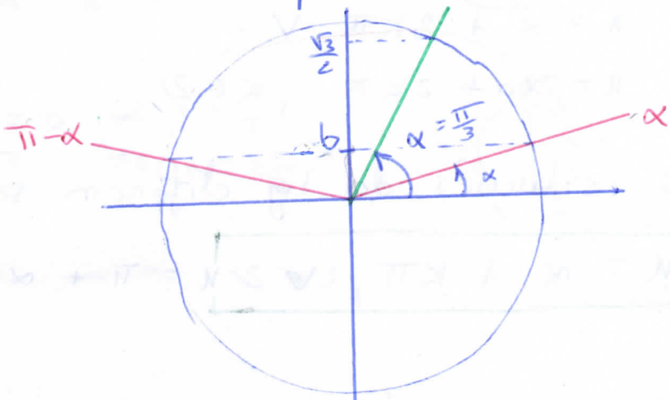
$C.S \neq \emptyset$



$\operatorname{sen} x = b$

$\downarrow$

$\operatorname{sen} \alpha = b$



$\Leftrightarrow \operatorname{sen} x = \operatorname{sen} \alpha$

radianos $\begin{cases} \Leftrightarrow x = \alpha + k \times 2\pi, \ k \in \mathbb{Z} \ \vee \\ x = \pi - \alpha + 2k\pi, \ k \in \mathbb{Z} \end{cases}$

graus $\{ \Leftrightarrow x = \alpha + k \times 360^\circ \vee x = 180^\circ - \alpha + k \times 360^\circ, k \in \mathbb{Z}$

$\operatorname{sen} x = \frac{\sqrt{3}}{2} \Leftrightarrow x = \frac{\pi}{3}$

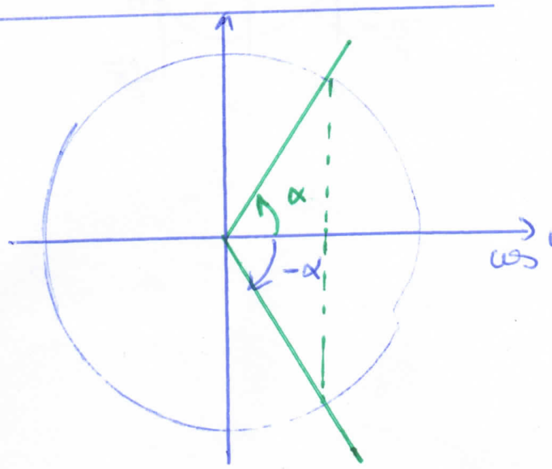
$\operatorname{sen} x = 0,3 \Leftrightarrow x = \operatorname{sen}^{-1}(0,3)$

$\cos x = b$

$-1 \leq b \leq 1$

$\downarrow$

$\cos x = b$

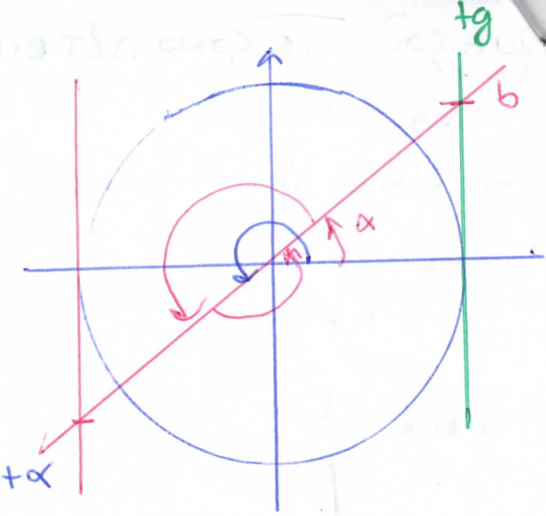


$\Leftrightarrow \cos x = \cos \alpha$

$\Leftrightarrow x = \alpha + 2k\pi \ \vee \ x = -\alpha + 2k\pi, \ k \in \mathbb{Z}$

$x = \alpha + k \times 360^\circ \ \vee \ x = -\alpha + k \times 360^\circ, \ k \in \mathbb{Z}$

EQUAÇÕES $tg\,x = b$



$\Leftrightarrow tg\,x = tg\,\alpha$

$\Leftrightarrow x = \alpha + 2k\pi \;\vee$

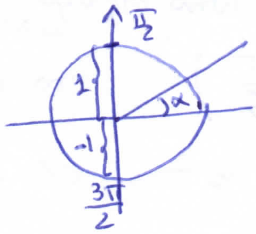
$x = \pi + \alpha + 2k\pi \;, k \in \mathbb{Z}$

As soluções da tg diferem sempre π, ao escrever $k\pi$ já estou a escrever α e $\pi + \alpha$

$$\boxed{x = \alpha + k\pi, k \in \mathbb{Z}}$$

$x = \alpha + 180°k \;, k \in \mathbb{Z}$

Casos particulares



$\sin x = 1$

$\Rightarrow x = \frac{\pi}{2} + 2k\pi, \; k \in \mathbb{Z}$

$k = 0 \quad x = \frac{\pi}{2}$

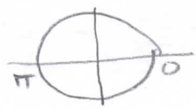
$k = 1 \quad x = \frac{5\pi}{2}$

$\rightarrow \sin x = 0$

$x = k\pi, \; k \in \mathbb{Z}$

$k = 0 \Rightarrow x = 0$

$k = 1 \Rightarrow x = \pi$

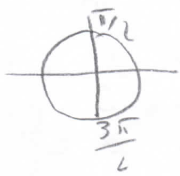


$\cos x = 0$

$x = \frac{\pi}{2} + k\pi; \; k \in \mathbb{Z} \Rightarrow$

$k = 0 \Rightarrow x = \frac{\pi}{2}$

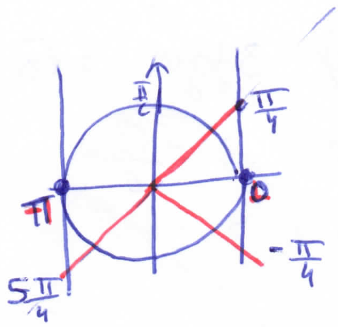
$k = 1 \Rightarrow x = \frac{3\pi}{2}$



$\cos x = 1$

$x = 0 + 2k\pi \vee x = \pi + 2k\pi$

$\boxed{tg\,x}$



$tg\,x = 1 \quad \vee \quad tg\,x = -1$

$tg\,x = tg\frac{\pi}{4} \quad \vee \quad tg\,x = -\frac{\pi}{4}$

$x = \frac{\pi}{4} + k\pi \quad \vee \quad x = -\frac{\pi}{4} + k\pi, \; k \in \mathbb{Z}$

$tg\,x = 0$

$\Rightarrow x = k\pi, \; k \in \mathbb{Z}$

$k = 0 \Rightarrow x = 0$

$k = 1 \Rightarrow x = \pi$

$P = \frac{2\pi}{|k|} = \frac{2\pi}{2} = \pi$

ex 78.2) L. 11º Novo espaço

$tg\left(-\frac{x}{2}\right) = tg\left(\frac{3\pi}{5}\right)$, em $\mathbb{R}$

$\Leftrightarrow -\frac{x}{2} = \frac{3\pi}{5} + k\pi, \ k \in \mathbb{Z}$

$\Leftrightarrow x = -\frac{3\pi}{5} \times 2 + 2k\pi, \ k \in \mathbb{Z}$

$\Leftrightarrow x = -\frac{6\pi}{5} - 2k\pi, \ k \in \mathbb{Z}$

ex 78.3) $tg(-x) = tg\,\frac{x}{2}$, em $\mathbb{R}$

$-x = \frac{x}{2} + k\pi, \ k \in \mathbb{Z}$

$\Leftrightarrow -x - \frac{x}{2} = k\pi, \ k \in \mathbb{Z}$

$\Leftrightarrow -\frac{3x}{2} = k\pi, \ k \in \mathbb{Z} \Leftrightarrow x = -\frac{2}{3}k\pi, \ k \in \mathbb{Z}$

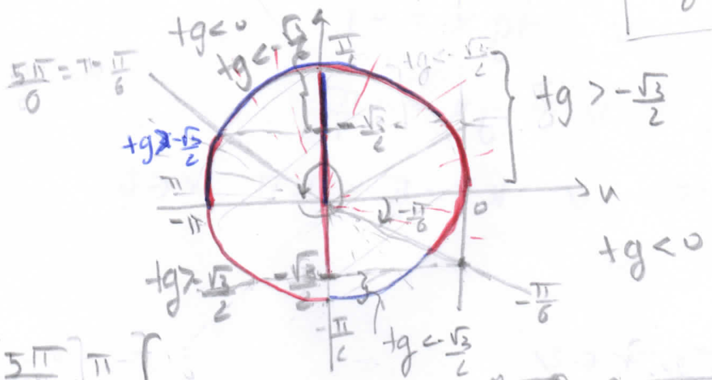
80.2) $3\,tg\,x + \sqrt{3} \geq 0 \ \wedge \ x \in \left]-\pi, \pi\right[$

$tg\,\frac{\pi}{3} = \sqrt{3}$

$tg\,\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

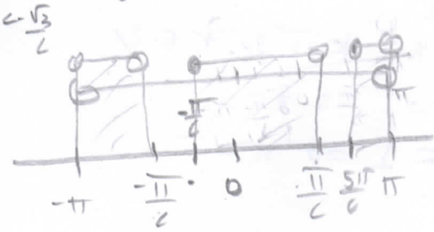
$\Leftrightarrow tg\,x \geq -\frac{\sqrt{3}}{3}$

$\Leftrightarrow tg\,x \geq tg\left(-\frac{\pi}{6}\right)$



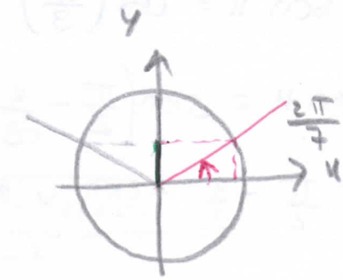
$x \in \left]-\pi, \pi\right[$

$\left]-\pi, -\frac{\pi}{2}\right[\cup \left[-\frac{\pi}{6}, \frac{\pi}{2}\right[\cup \left[\frac{5\pi}{6}, \pi\right[$



Soluções equações trigonométricas

1. $\operatorname{sen} x = \operatorname{sen} \frac{2\pi}{7}$



$\Leftrightarrow \square = \Delta + 2k\pi$

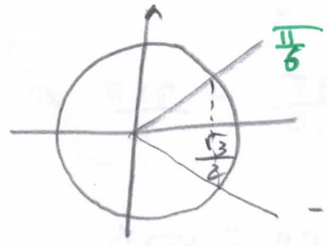
$\Leftrightarrow \square = \Delta + 2k\pi \vee \square = \pi - \Delta + 2k\pi,\ k \in \mathbb{Z}$

$\sin(\square) = \sin(\Delta)$

$\Leftrightarrow x = \frac{2\pi}{7} + 2k\pi \vee x = \pi - \frac{2\pi}{7} + 2k\pi,\ k \in \mathbb{Z}$

$\Leftrightarrow x = \frac{2\pi}{7} + 2k\pi \vee x = \frac{5\pi}{7} + 2k\pi,\ k \in \mathbb{Z}$

1.2) $\cos\left(x + \frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$



$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

$\Leftrightarrow x + \frac{\pi}{3} = \frac{\pi}{6} + 2k\pi \vee x + \frac{\pi}{3} = -\frac{\pi}{6} + 2k\pi,\ k \in \mathbb{Z}$

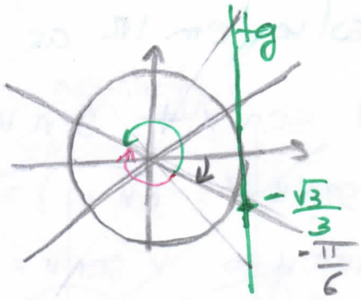
$\Leftrightarrow x = \frac{\pi}{6} - \frac{\pi}{3} + 2k\pi \vee x = -\frac{\pi}{6} - \frac{\pi}{3} + 2k\pi,\ k \in \mathbb{Z}$

$\Leftrightarrow x = -\frac{\pi}{6} + 2k\pi \vee x = \frac{-3\pi}{6} + 2k\pi,\ k \in \mathbb{Z}$

$\Leftrightarrow x = -\frac{\pi}{6} + 2k\pi \vee x = -\frac{\pi}{2} + 2k\pi,\ k \in \mathbb{Z}$

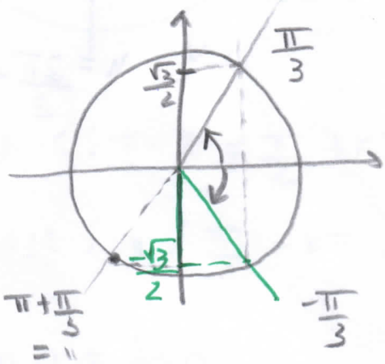
1.3) $\operatorname{tg} x = -\frac{\sqrt{3}}{3}$

$x = -\frac{\pi}{6} + k\pi,\ k \in \mathbb{Z}$



pode andar $k\pi$ no sentido positivo ou negativo

2.1) $4 \operatorname{sen}\left(\frac{\pi}{3} - 2x\right) + 2\sqrt{3} = 0$



$\Leftrightarrow 4 \operatorname{sen}\left(\frac{\pi}{3} - 2x\right) = -2\sqrt{3} \Leftrightarrow$

$\Leftrightarrow \operatorname{sen}\left(\frac{\pi}{3} - 2x\right) = -\frac{2\sqrt{3}}{4} \Leftrightarrow$

$\Leftrightarrow \operatorname{sen}\left(\frac{\pi}{3} - 2x\right) = -\frac{\sqrt{3}}{2}$

$\Leftrightarrow \frac{\pi}{3} - 2x = -\frac{\pi}{3} + 2k\pi \vee \frac{\pi}{3} - 2x = \pi - \left(-\frac{\pi}{3}\right) + 2k\pi,\ k \in \mathbb{Z}$

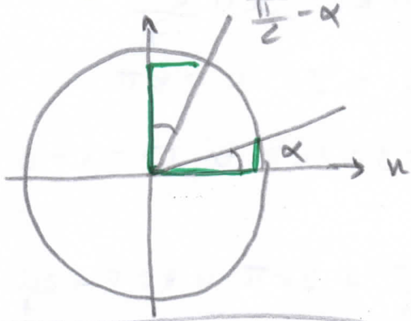
$\Leftrightarrow -2x = -\frac{\pi}{3} - \frac{\pi}{3} + 2k\pi \vee -2x = \pi + \frac{\pi}{3} - \frac{\pi}{3} + 2k\pi,\ k \in \mathbb{Z}$

$\Leftrightarrow x = \frac{-\frac{2\pi}{3}}{-2} + \frac{2k\pi}{-2} \vee x = \frac{\pi}{-2} + \frac{2k\pi}{-2},\ k \in \mathbb{Z} \Leftrightarrow x = \frac{\pi}{3} - k\pi \vee x = -\frac{\pi}{2} - k\pi,\ k \in \mathbb{Z}$

2.2) $\operatorname{sen} x = \cos\left(\frac{x}{3}\right)$

Nota

$\sin\left(\frac{\pi}{2} - \alpha\right) = \cos\alpha$



$\Leftrightarrow \operatorname{sen} x = \operatorname{sen}\left(\frac{\pi}{2} - \frac{x}{3}\right)$

$\Leftrightarrow x = \frac{\pi}{2} - \frac{x}{3} + 2k\pi \vee x = \pi - \left(\frac{\pi}{2} - \frac{x}{3}\right) + 2k\pi, k \in \mathbb{Z}$

$\Leftrightarrow x + \frac{x}{3} = \frac{\pi}{2} + 2k\pi \vee x - \frac{x}{3} = \frac{2\pi - \pi}{2} + 2k\pi, k \in \mathbb{Z}$

$\Leftrightarrow \frac{4x}{3} = \frac{\pi}{2} + 2k\pi \vee \frac{3x - x}{3} = \frac{\pi}{2} + 2k\pi, k \in \mathbb{Z}$

$\boxed{\sin\left(\frac{\pi}{2} - \frac{x}{3}\right) = \cos\left(\frac{x}{3}\right)}$

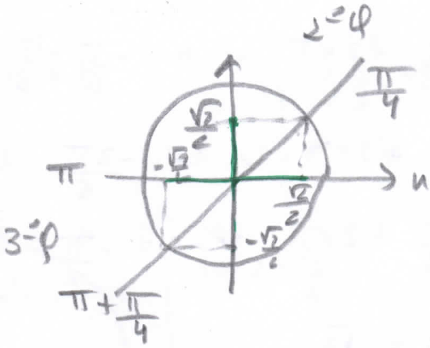
$\Leftrightarrow x = \frac{\frac{3\pi}{2}}{4} + 6k\pi \vee \frac{2x}{3} = \frac{\pi}{2} + 2k\pi, k \in \mathbb{Z}$

$\Leftrightarrow x = \frac{3\pi}{8} + \frac{3k\pi}{2} \vee x = \frac{3\pi}{4} + 3k\pi, k \in \mathbb{Z}$

2.3) $\operatorname{sen} = \cos x$

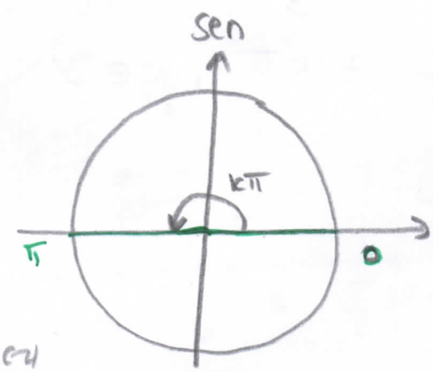
$\Leftrightarrow x = \frac{\pi}{4} + 2k\pi \vee x = \pi + \frac{\pi}{4} + 2k\pi, k \in \mathbb{Z}$

$\Leftrightarrow \boxed{x = \frac{\pi}{4} + k\pi, k \in \mathbb{Z}}$



Resolve em IR as equações em IR

3.1) $\operatorname{sen}^2 x + \operatorname{sen} x = 0$



$\Leftrightarrow \operatorname{sen} x(\operatorname{sen} x + 1) = 0 \Leftrightarrow$

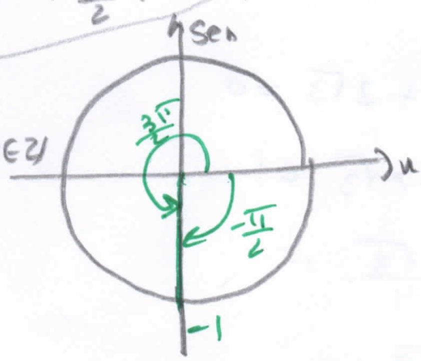
$\Leftrightarrow \operatorname{sen} x = 0 \vee \operatorname{sen} x = -1$

$\Leftrightarrow \operatorname{sen} x = \operatorname{sen} 0 \vee \operatorname{sen} x = \operatorname{sen}\frac{3\pi}{2}$

$\Leftrightarrow x = 0 + k\pi \vee x = \frac{3\pi}{2} + 2k\pi \vee x = \pi - \frac{3\pi}{2} + 2k\pi, k \in \mathbb{Z}$

$\Leftrightarrow x = k\pi$

$\vee \quad x = \frac{3\pi}{2} + 2k\pi, k \in \mathbb{Z}$



Nota

$\operatorname{sen} x = \frac{1}{2}$

$\operatorname{sen} x = \operatorname{sen} \cancel{\frac{1}{2}}$

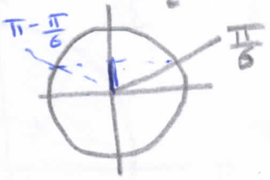
$\operatorname{sen} x = \operatorname{sen}\frac{\pi}{6}$

$\frac{3\pi}{2} = -\frac{\pi}{2}$

$$2\,\text{sen}^2 x + \text{sen}\, x - 1 = 0$$

$\Leftrightarrow \text{sen}\, x = \frac{1}{2} \vee \text{sen}\, x = -1$

$\Leftrightarrow \text{sen}\, x = \text{sen}\frac{\pi}{6} \vee \text{sen}\, x = \text{sen}\, \frac{3\pi}{2}$



$\Leftrightarrow x = \frac{\pi}{6} + 2k\pi \vee x = \pi - \frac{\pi}{6} + 2k\pi \vee x = \frac{3\pi}{2} + k\pi$, $k \in \mathbb{Z}$

m.v.

$\text{sen}\, x = y$

$2y^2 + y - 1 = 0$

$\Leftrightarrow y = \frac{-1 \pm \sqrt{1^2 - 4 \times 2 \times (-1)}}{2 \times 2}$

$\Leftrightarrow y = \frac{-1 \pm 3}{4} \Leftrightarrow y = \frac{-1+3}{4} \vee y = \frac{-1-3}{4}$

$\Leftrightarrow y = \frac{1}{2} \vee y = -1$

$\Leftrightarrow x = \frac{\pi}{6} + 2k\pi \vee x = \frac{5\pi}{6} + 2k\pi \vee x = \frac{3\pi}{2} + k\pi$, $k \in \mathbb{Z}$

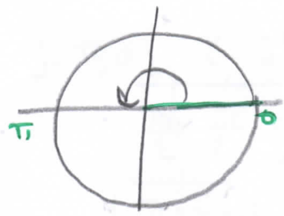
4.1) $\text{sen}^4 x + \text{sen}^2 x \times \cos^2 x = 0$

$\Leftrightarrow \text{sen}^2 x \times \text{sen}^2 x + \text{sen}^2 x \times \cos^2 x = 0$

$\Leftrightarrow \text{sen}^2 x\, (\text{sen}^2 x + \cos^2 x) = 0$

$\Leftrightarrow \text{sen}^2 x \times 1 = 0 \Leftrightarrow \text{sen}^2 x = 0$

$\Leftrightarrow \text{sen}\, x = \pm\sqrt{0} \Leftrightarrow \text{sen}\, x = 0$



$\Leftrightarrow \text{sen}\, x = \text{sen}\, 0$

$\Leftrightarrow x = 0 + k\pi$, $k \in \mathbb{Z}$ $\Leftrightarrow \boxed{x = k\pi,\ k \in \mathbb{Z}}$

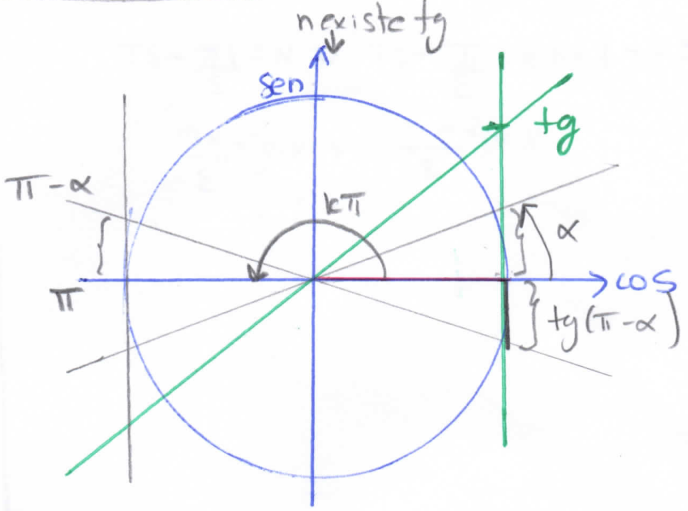
4.2) $\text{sen}\, x + \text{tg}(\pi - x) = 0$

$\Leftrightarrow \text{sen}\, x - \text{tg}\, x = 0$

$\Leftrightarrow \text{sen}\, x = \text{tg}\, x$

$\Leftrightarrow x = 0 + k\pi$, $k \in \mathbb{Z}$

$\Leftrightarrow x = k\pi$, $k \in \mathbb{Z}$



ou

$\Leftrightarrow \text{sen}\, x = \frac{\text{sen}\, x}{\cos x}$

$\Leftrightarrow \text{sen}\, x \cdot \cos x = \text{sen}\, x$

$\Leftrightarrow \text{sen}\, x\, (\cos x - 1) = 0$

$\Leftrightarrow \text{sen}\, x = 0 \vee \cos x = 1$

$\Leftrightarrow x = k\pi \vee x = 2k\pi$, $k \in \mathbb{Z}$

$D = \{x \in \mathbb{R} : \pi - x \neq \frac{\pi}{2} + k\pi,\ k \in \mathbb{Z}\}$

$= \{x \in \mathbb{R} : x \neq \frac{\pi}{2} - k\pi,\ k \in \mathbb{Z}\}$

$\text{sen}\, 0 = \text{tg}\, 0$

Resolve as equações no intervalo indicado.

5.1. $4\,sen\,x - 2\sqrt{3} = 0$ em $\left]\frac{\pi}{2}, \frac{3\pi}{2}\right[$

1º processo

$\Leftrightarrow 4\,sen\,x = 2\sqrt{3}$

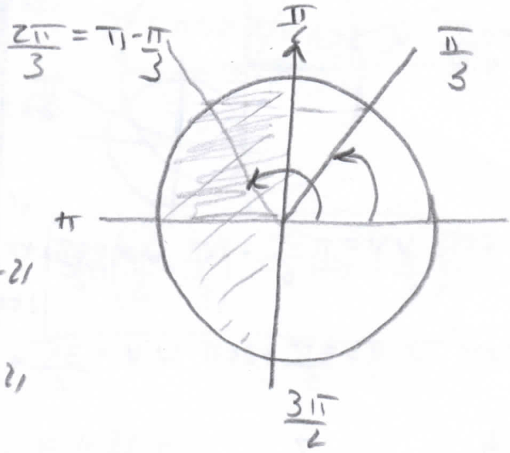
$\Leftrightarrow sen\,x = \frac{\sqrt{3}}{2}$

$\Leftrightarrow sen\,x = sen\,\frac{\pi}{3}$

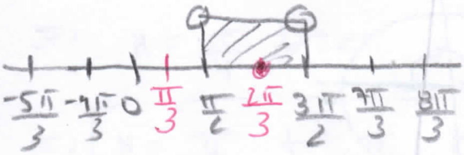
$\Leftrightarrow x = \frac{\pi}{3} + 2k\pi \vee x = \pi - \frac{\pi}{3} + 2k\pi, k \in \mathbb{Z}$

$\Leftrightarrow x = \cancel{\frac{\pi}{3} + 2k\pi} \vee x = \frac{2\pi}{3} + 2k\pi, k \in \mathbb{Z}$

$C.S = \left\{\frac{2\pi}{3}\right\}$



2º processo



$k = 0 \rightarrow \cancel{x = \frac{\pi}{3}} \vee \boxed{x = \frac{2\pi}{3}}$

$k = 1 \rightarrow \cancel{x = \frac{\pi}{3} + 2\pi = \frac{7\pi}{3}} \vee \cancel{x = \frac{2\pi}{3} + 2\pi = \frac{8\pi}{3}}$

$k = -1 \rightarrow x = \frac{\pi}{3} - 2\pi \vee x = \frac{2\pi}{3} - 2\pi$

$\cancel{x = -\frac{5\pi}{3}} \vee \cancel{x = -\frac{4\pi}{3}}$

3º processo

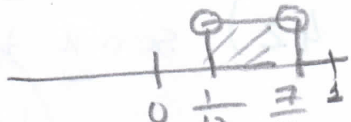
$\frac{\pi}{2} < x < \frac{3\pi}{2}$

$\frac{\pi}{2} < \frac{\pi}{3} + 2k\pi < \frac{3\pi}{2}$

$\Leftrightarrow \frac{\pi}{2} - \frac{\pi}{3} < 2k\pi < \frac{3\pi}{2} - \frac{\pi}{3}$

$\Leftrightarrow \frac{\frac{\pi}{6}}{2\pi} < k < \frac{\frac{7\pi}{6}}{2\pi}$

$\Leftrightarrow \frac{1}{12} < k < \frac{7}{12}$



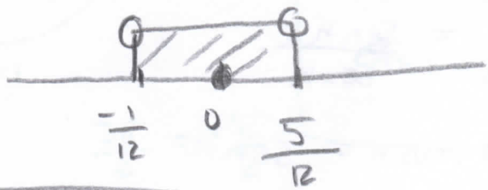
e

$\frac{\pi}{2} < x < \frac{3\pi}{2}$

$\frac{\pi}{2} < \frac{2\pi}{3} + 2k\pi < \frac{3\pi}{2}$

$\Leftrightarrow -\frac{\pi}{6} < 2k\pi < \frac{5\pi}{6}$

$\Leftrightarrow -\frac{1}{12} < k < \frac{5}{12}$



$\boxed{k = 0}$